Graphs of functions - Quadratics

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1. Consider the data points presented in the table below. Plot each data set (x, y_n) on separate axes. The horizontal axis should cover the interval $-10 \leq x \leq 10$, and the vertical axis $-10 \leq y \leq 10$. Make sure to show a clear scale and label for each axis.

x	y_1	y_2	y_3	y_4	y_5
-4.00	-35	30	0.0625	0	-0.25
-3	-12	20	0.125	1	$-0.\overline{3}$
-2	0	12	0.25	1.41	-0.5
-1	4	6	0.5	1.73	-1
0	3	2	1	2	$-\infty$
1	0	0	2	2.23	1
2	-2	0	4	2.45	0.5
3	0	2	8	2.64	$0.\overline{3}$
4	9	6	16	2.83	0.25

2. Match each of your graphs to one of the functions below. Write down any crucial characteristics of each kind of function.

   - $f(x) = \frac{1}{x}$
   - $f(x) = (x-1)(x-2)$
   - $f(x) = \sqrt{x+4}$
   - $f(x) = \frac{1}{2}(x-1)(x+2)(x-3)$
   - $f(x) = 2^x$

3. which of the functions in 2. are one-to-one? which ones are many-to-one?

4. The data shown in the table below corresponds to data collected of the height of a projectile in vertical motion.

5. Calculate the first and second differences of the heights. What do you notice?

6. Graph the points and connect them with a smooth curve.

Time t	Height y
0	0
1	35
2	60
3	75
4	80
5	75
6	60
7	35
8	0

7. Consider the function $f(t) = at^2 + bt + c$. Your data can be modeled through a quadratic function just like $f(x)$. By substituting the values of three data points of your choice into $f(x)$, create a system of linear equations and find the values of a, b, and c.
8. Use your values of a, b, and c to write an expression for $f(t)$.
9. What is the maximum height of the projectile?
10. What is the time at which the projectile reaches its maximum value.
11. Write an expression for $f(t)$ showing the vertex of this function.
12. Write an expression for $f(t)$ showing the starting and ending times of the trajectory, explicitly.
13. Why is it important to use different notations for a function like $f(t)$?
14. Consider the general form $f(t) = at^2 + bt + c$. Using the method of completing the square, show that this can also be written as $f(t) = a(x - h)^2 + k$.
15. Hence, write down an expression for the x coordinate, h, of the vertex (h, k).
16. What does the vertical line $x = h$ represent?
17. Using technology, graph different version s of the function $f(t) = a(x - h)^2 + k$, keeping the values of h and k fixed, and changing the values of a. What is the effect of the parameter a? When is the function concave up? When is it concave down?