EXERCISES 3.1-3.3

TRIANGLES - ARCS AND SECTORS

Compiled by Christos Nikolaidis

A. Practice Questions

1. In a triangle *ABC,* $A\hat{B}C$ *=30°, AB=6cm , AC=* $3\sqrt{2}$ *cm.* Find the possible areas of the triangle.

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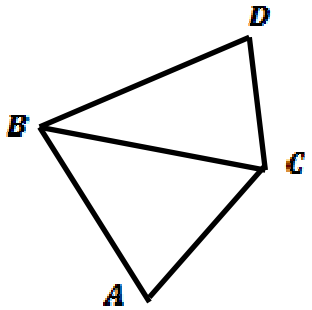
2. Consider the following diagram



$AB = 7$

$AC = 5$

$\hat{A} = 60^\circ$

$\hat{D} = 80^\circ$

$D\hat{B}C = 30^\circ$

Find

(a) the length of the side BD.

(b) the area of the quadrilateral $ABDC$.

(c) the perimeter of the quadrilateral $ABDC$.

It is given that the bearing from B to D is 70^o. Find

(d) the bearing from B to A.

(e) the bearing from A to B.

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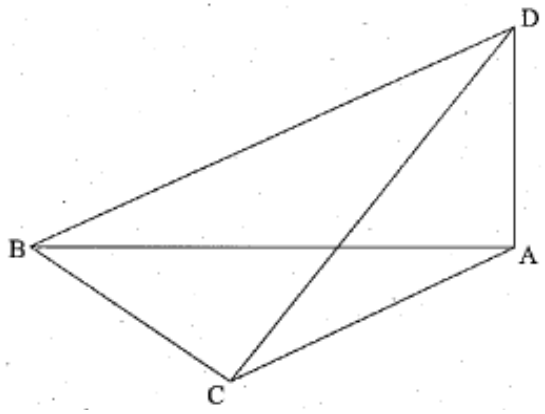
3. The lengths of two sides in a triangle are 5 cm and 7 cm respectively, while one of the angles is 40 degrees. Find **all** possible lengths of the third side.

4. The following three dimensional diagram shows the four points A,B,C and D.
A,B,C are in the same horizontal plane and AD is vertical. The ABC is 45° , and BC = 50m.
The angle of elevation from point B to point D is 30°, while the angle of elevation from point C to point D is 20°.



Using the cosine rule in the triangle ABC, or otherwise, find AD.

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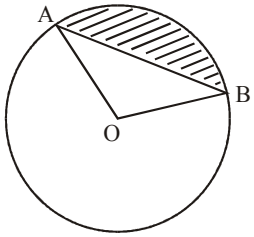
5. The diagram shows a circle of centre O and radius r. The angle AOB is θ radians.



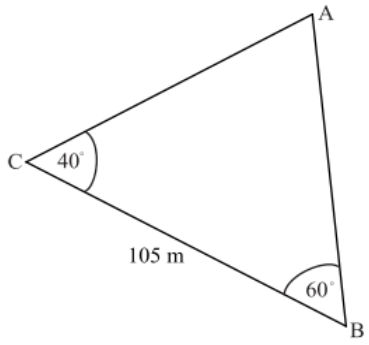
Find the value of θ, in each of the following two cases

(a) given that the ratio of the length of the minor arc AB to the length of the major arc AB is 1:5.

(b) given that the ratio between the areas of the shaded region to the non-shaded region in the circle is 1:5.

B. Past Paper questions (*SHORT*)

6. The following diagram shows ΔABC, where $BC = 105$ m, $A\hat{C}B = 40^\circ$, $A\hat{B}C = 60^\circ$



Find AB

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(Total 6 marks)

7. In the triangle ABC, $\hat{A} = 30°$, BC = 3 and AB = 5. Find the two possible values of $\hat{B}$.

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(Total 6 marks)

8. In a triangle ABC, $\hat{A} = 35^\circ$, $BC = 4$ cm and $AC = 6.5$ cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

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(Total 7 marks)

Notice: In the following case you may use either Sine or Cosine rule. Please follow both methods.

9. In a triangle ABC, $A\hat{B}C = 30°$, AB = 6cm , AC = $3\sqrt{2}$ cm. Find the possible lengths of [BC].

METHOD A: Use Sine rule.

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METHOD B: Use Cosine rule.

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(Total 6 marks)

10. In triangle ABC, $A\hat{B}C = 31°$, AC = 3cm , BC = 5cm. Calculate the possible lengths of [AB].

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(Total 6 marks)

11. Consider triangle ABC with $B\hat{A}C = 37.8^{o}$, AB = 8.75 and BC = 6. Find AC.

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(Total 7 marks)

12. Triangle ABC has AB = 8 cm, BC = 6 cm, $B\hat{A}C = 20°$. Find the smallest possible area of ΔABC.

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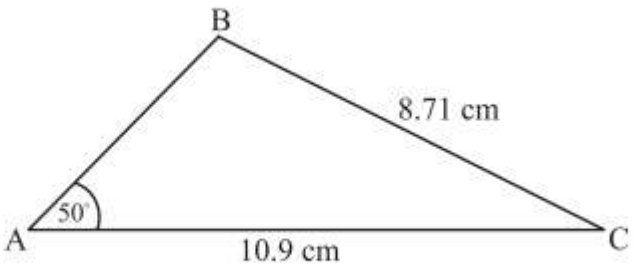
(Total 6 marks)

13. In the **obtuse-angled** triangle ABC, $AC = 10.9$ cm, $BC = 8.71$ cm and $B\hat{A}C = 50^\circ$.

Not to scale



Find the area of triangle ABC.

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(Total 6 marks)

14. Triangle ABC has $\hat{C} = 42^\circ$, $BC = 1.74$ cm, and area 1.19 cm^2.

(a) Find AC.

(b) Find AB.

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(Total 6 marks)

15. In the triangle ABC, $\hat{A} = 30^\circ$, $a = 5$ and $c = 7$. Find the difference in area between the two possible triangles for ABC.

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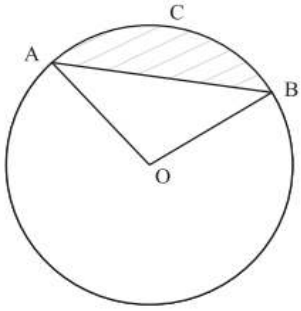
(Total 6 marks)

16. The following diagram shows a circle centre O and radius r. The length of the arc ACB is $2r$.



The area of the shaded segment may be expressed as kr^2. Find the value of k.

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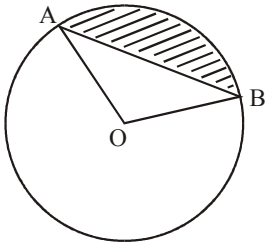
(Total 6 marks)

17. The diagram below shows a circle centre O and radius OA = 5 cm. The angle AOB = 135°.



Find the area of the shaded region.

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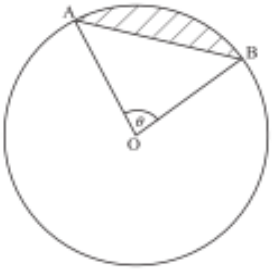
(Total 6 marks)

18. The diagram shows a circle centre O and radius 1, with $A\hat{O}B = \theta$, $\theta \neq 0$. The area of ΔAOB is three times the shaded area.



Find the value of θ

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(Total 6 marks)

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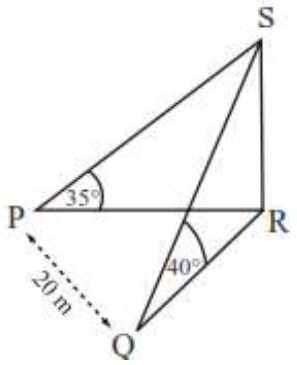

The above three dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively 35° and 40°, and $PQ = 20$ m.

Determine the height of the flagpole.

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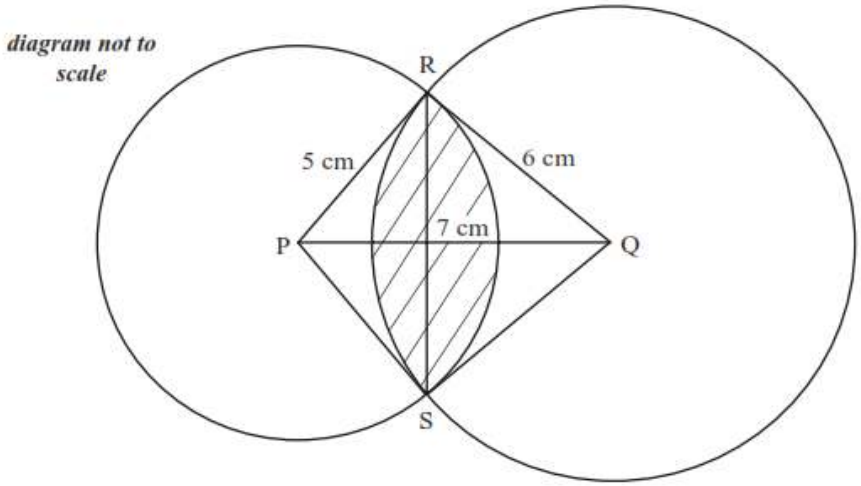
(Total 8 marks)

20. The diagram below shows a pair of intersecting circles with centres at P and Q with radii of 5 cm and 6 cm respectively. RS is the common chord of both circles and PQ is 7 cm.



Find the area of the shaded region.

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(Total 6 marks)

21. Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs. This is shown in the diagram below.

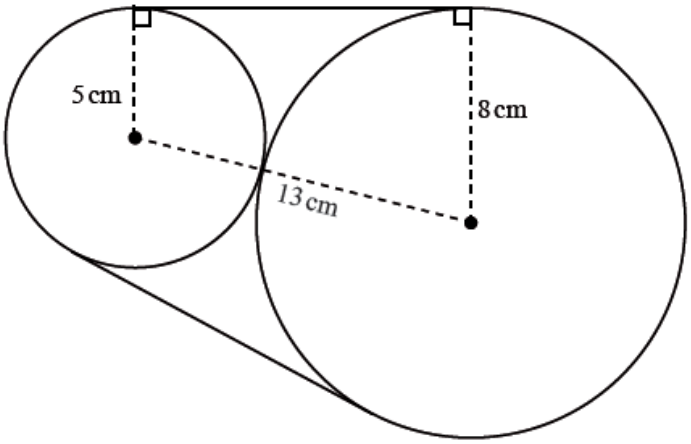

diagram not to scale

Calculate the length of the string needed to go around the discs.

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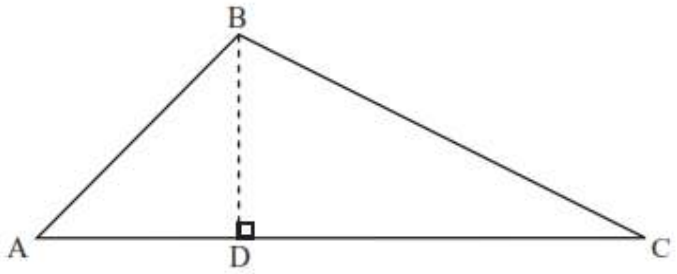
(Total 8 marks)

C. Past Paper Questions (*LONG*)

(Please answer on separate answer sheets)

22. In triangle ABC, $BC = a$, $AC = b$, $AB = c$ and [BD] is perpendicular to [AC].



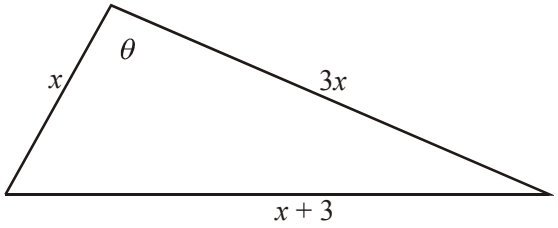
(a) Show that $CD = b - c\cos A$. *[1 mark]*

(b) **Hence**, by using Pythagoras' Theorem in the triangle BCD, prove the cosine rule for the triangle ABC. *[4 marks]*

(c) If $A\hat{B}C = 60^\circ$, use the cosine rule to show that $c = \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$. *[7 marks]*

(Total 12 marks)

23. The area of the triangle shown below is 2.21 cm^2. The length of the shortest side is x cm and the other two sides are $3x$ cm and $(x + 3)$ cm.



(a) Using the formula for the area of the triangle, write down an expression for $\sin\theta$ in terms of x.

(2)

(b) Using the cosine rule, write down and simplify an expression for $\cos\theta$ in terms of x.

(2)

(c) (i) Using your answers to parts (a) and (b), show that,

$$\left(\frac{3x^2 - 2x - 3}{2x^2}\right)^2 = 1 - \left(\frac{4.42}{3x^2}\right)^2$$

(1)

(ii) Hence find

(a) the possible values of x;

(2)

(b) the corresponding values of θ, **in radians,** using your answer to part (b) above.

(3)

(Total 10 marks)

24. A farmer owns a triangular field ABC. The side [AC] is 104 m, the side [AB] is 65 m and the angle between these two sides is 60°.

(a) Calculate the length of the third side of the field. **(3)**

(b) Find the area of the field in the form $p\sqrt{3}$, where p is an integer. **(3)**

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts by constructing a straight fence [AD] of length x metres.

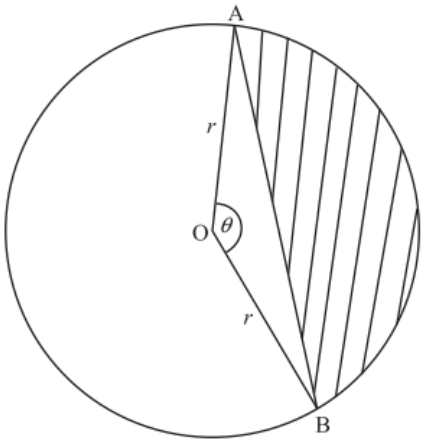
(c) (i) Show that the area of the smaller part is given by $\frac{65x}{4}$ and find an expression for the area of the larger part.

(ii) Hence, find the value of x in the form $q\sqrt{3}$, where q is an integer. **(8)**

(d) Prove that $\frac{BD}{DC} = \frac{5}{8}$. **(6)**

(Total 20 marks)

25. The following diagram shows a circle centre O, radius r. The angle $A\hat{O}B$ at the centre of the circle is θ radians. The chord AB divides the circle into a minor segment (the shaded region) and a major segment.



(a) Show that the area of the minor segment is $\frac{1}{2}r^2(\theta - \sin\theta)$. *[4 marks]*

(b) Find the area of the major segment. *[3 marks]*

(c) Given that the ratio of the areas of the two segments is 2:3, show that $\sin\theta = \theta - \frac{4\pi}{5}$. *[4 marks]*

(d) Hence find the value of θ. *[2 marks]*