Investigation - Functions Lesson 1
Linear functions

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This investigation will walk you through the concepts of domain and range and linear functions.

1. Consider the set of values $X_a = \{x|x = 1, 2, 3, 4, 5\}$ and $Y_a = \{1, 4, 9, 16, 25\}$. What is the relationship between Sets X_a and Y_a? Use a mapping to illustrate your answer.

2. Consider the set of values $X_b = \{x|x = 1, 2, 3, 4, 5\}$ and $Y_b = \{1, 3, 5, 7, 9\}$. What is the relationship between Sets X_b and Y_b? Use a mapping to illustrate your answer.

3. Consider the set of values $X_c = \{x|x = -3, -2, -1, 0, 1, 2, 3\}$ and $Y_c = \{9, 4, 1, 0, 1, 4, 9\}$. What is the relationship between Sets X_c and Y_c? Use a mapping to illustrate your answer.

4. Compare the relation between sets X_a and Y_a, and sets X_c and Y_c. How would you describe the difference?

5. A function is one-to-one (injective function) if each element in the co-domain is the image of at most one element in the domain. From the functions in questions 1 3, which are injective functions?

6. In Geogebra (https://www.geogebra.org/classic), graph the relation $f : x \in X_b \rightarrow y \in Y_b$.

7. Draw the line that goes through all of the given points and write down the equation of the line.

8. Draw the line y=0, which corresponds to the horizontal axis.

9. use the tool to measure the angle that goes from the x axis, to the graph of the function b.

   Choosing any two points on the function $f : x \in X_b \rightarrow y \in Y_b$, calculate the rate of change (slope/gradient) given by the ratio between the change vertically $(y_2 - y_1)$ and the change horizontally $(x_2 - x_1)$. Next compare this value to the value of the tangent of the angle you found in 8.

10. Repeat 6 9 for the following relations, then complete the table below.

    - $X_d = \{x|x = 3, 6, 9, 12\}$ and $Y_d = \{-11, -2, 7, 16\}$
    - $X_e = \{x|x = -2, -1, 0, 1, 2, 3\}$ and $Y_e = \{2, \frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}\}$
    - $X_f = \{x|x = -3, -1, 1, 3, 5\}$ and $Y_f = \{-1, -3, -5, -7, -9\}$
    - $X_g = \{x|x = -4, -2, 0, 2, 4, 6, 8\}$ and $Y_g = \{-2, -1, 0, 1, 2, 3, 4\}$

Function	Equation of the line	Angle, α	$\tan\alpha$	slope/Gradient
b				
d				
e				
f				
g				

11. From the results on your table, what observations can you make regarding the relationship between the angle α and the gradient/slope?

12. Describe mathematically the situations in which one would have an increasing linear function, that is that goes up from left to right? And when would it be decreasing, that is that goes down fro left to right?

13. What is the value of $\tan\alpha$ when the linear function is completely horizontal? How would you describe the change of the function in that situation?

14. What is the value of $\tan\alpha$ when the linear function is completely vertical? IS is a function? How would you write an equation for this type of linear relation?

15. There are several ways in which you can express the equation of a line:

    - Slope-Intercept form: $y = mx + c$.
    - Cartesian form: $ax + by + c = 0$.
    - Point-Slope form: $y - y_1 = m(x - x_1)$.
    - Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$, where a is the $x-$intercept and b is the $y-$intercept.

    Explore the different ways of writing the equation of a line by transforming the expressions given by Geogebra in Slope-intercept form for the functions in your table, into each of the other forms of the linear function presented above, by algebraic manipulation or otherwise.