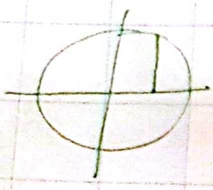
$= \quad = \sec^2 x$

$$x = \frac{3}{2}\tan\theta \qquad \int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{3/2}}\,dx = \int_0^{\pi/3} \frac{\left(\frac{3}{2}\right)^3 \tan^3\theta}{\left(4\left(\frac{3}{2}\right)^2\tan^2\theta + 9\right)^{3/2}} \cdot \frac{3}{2}\sec$$

$$dx = \frac{3}{2}\sec^2\theta \; d\theta.$$



$$\frac{3\sqrt{3}}{2} = \frac{3}{2}\tan\theta_1 \qquad 0 = \frac{3}{2}\tan\theta_2$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} = \tan\theta_1 \qquad \theta_2 = 0.$$

$$\theta_1 = \frac{\pi}{3}$$

$$\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{3/2}}\,dx = \int_0^{\pi/3} \frac{\left(\frac{3}{2}\right)^4 \tan^3\theta \sec^2\theta}{(9)^{3/2}(\tan^2\theta+1)^{3/2}}\,d\theta$$

$$= \frac{\left(\frac{3}{2}\right)^4}{3^3}\int_0^{\pi/3} \frac{\tan^3\theta \cancel{\sec^2\theta}}{\sec^{\cancel{3}}\theta}\,d\theta$$

$$= 3\left(\frac{1}{2}\right)^4 \int_0^{\pi/3} \frac{\tan^3\theta}{\sec\theta}\,d\theta \quad \checkmark$$

$$= 3\left(\frac{1}{2}\right)^4 \int_0^{\pi/3} \frac{\sin^3\theta}{\cos^3\theta \cdot \frac{1}{\cos\theta}}\,d\theta$$

$$= 3\left(\frac{1}{2}\right)^4 \int_0^{\pi/3} \frac{\sin^3\theta}{\cos^2\theta}\,d\theta$$

$$= 3\left(\frac{1}{2}\right)^4 \int_0^{\pi/3} \frac{(1-\cos^2\theta)}{\cos^2\theta}\sin\theta\,d\theta \qquad u = \cos\theta, \; du = -\sin\theta\,d\theta$$

$$= -3\left(\frac{1}{2}\right)^4 \int \frac{1-u^2}{u^2}\,du = -3\left(\frac{1}{2}\right)^4 \int (u^{-2}-1)\,du$$

$$= -3\left(\frac{1}{2}\right)^4 \left[-\frac{1}{u} - u\right] = -3\left(\frac{1}{2}\right)^4 \left[-\frac{1}{\cos\theta} - \cos\right.$$

$$= -3\left(\frac{1}{2}\right)^4 \left[-2 - \frac{1}{2} - (-1-1)\right] = 3\left(\frac{1}{2}\right)^5$$