4. Given that $\frac{1}{2}(-1+\sqrt{3}i)$ is a root of:

$3z^3+2z^2+2z-1 = 0$, find all other roots.

5. Given that $(z-1-2i)$ is a factor of $2z^3-3z^2+8z+5$ solve the equation $2z^3-3z^2+8z+5 = 0$ over the complex number field.

6. Given that $P(2-3i) = 0$, find all three linear factors of $z^3-7z^2+25z-39$.

7. Find all complex numbers, z, such that $z^4-z^3+6z^2-z+15 = 0$ and $z = 1+2i$ is a solution to the equation.

8. Factorise the following.

a $2z^3-z^2+2z-1$ b z^4+z^2-12

9. Given that $2-i$ is a root of $z^3+az^2+z+5 = 0$ where a is a real number, find all the roots to this equation.

10. Given that $2+3i$ is a root of $z^3+az^2+b = 0$, where a and b are real numbers, find all the roots of this equation.

11. Given that $2-i$ is a root of $2z^3-9z^2+14z-5 = 0$, find the other roots.

12. Given that $4-i$ is a zero of:

$P(z) = z^3+az^2+33z-34$,

find a and hence factorise $P(z)$.

14. Solve the following over the real number field.

a $z^6+7z^3-8 = 0$

b $z^6-9z^3+8 = 0$

c $z^4-2z^2-3 = 0$

d $z^4-4z^2-5 = 0$

15. Write down an equation of the lowest possible degree with real coefficients such that its roots are:

a $3, 2-i$

b $2, 1, 1+i$

c $1-\sqrt{3}i, 3$

d $1+\sqrt{2}i, -2+\sqrt{3}i$

16. Verify that $z = -1+\sqrt{3}i$ is a root of the equation $z^4-4z^2-16z-16 = 0$ and hence find the other roots.

17. Given that $z = a+ib$ is a root of $z^4-z^3-6z^2+11z+5 = 0$ and $Re(z) = 2$, solve the equation completely.

18. If $z^n+z^{-n} = 2\cos(n\theta)$ show that:

$5z^4-z^3-6z^2-z+5 = 0 \Rightarrow 10\cos^2\theta-\cos\theta-8 = 0.$

19. Show that $\cos 5\theta = 16\cos^5\theta-20\cos^3\theta+5\cos\theta$, and

hence show that the roots of $x(16x^4-20x^2+5) = 0$

are 0, $\cos\left(\frac{\pi}{10}\right)$, $\cos\left(\frac{3\pi}{10}\right)$, $\cos\left(\frac{7\pi}{10}\right)$, $\cos\left(\frac{9\pi}{10}\right)$.