1. [Maximum points: 13]

Find the particular solution to the equation

$$\frac{dy}{dx} = 0.4y\left(1 - \frac{y}{10}\right)$$

where $y(0) = 1$.

2. [Maximum points: 6]

Solve the differential equation $\frac{dy}{dx} - 1 = y^2$ where $y(0) = 0$.

1. Separate the variables A1

$$\int \frac{1}{y\left(1-\frac{y}{10}\right)}\,dy = \int 0.4\,dx$$

Rewrite the denominator using partial fractions M1

$$\frac{10}{y(10-y)} = \frac{a}{y} + \frac{b}{10-y}$$

Rearrange

$$10 = a(10-y) + by$$ A1

Equate coefficients M1

$$a = 1$$ A1

and

$$-1 + b = 0$$

So

$$b = 1$$ A1

We then have

$$\int \frac{1}{y} + \frac{1}{10-y}\,dy = \int 0.4\,dx$$

Integrate

$$\ln|y| - \ln|10-y| = 0.4x + c$$ M1

Rewrite

$$\frac{y}{10-y} = \pm A e^{0.4x}$$ A1

Use the initial conditions M1

$$\frac{1}{9} = \pm A$$ A1

So

$$\frac{y}{10-y} = \frac{e^{0.4x}}{9}$$ A1

Rearrange

$$y\left(9 + e^{0.4x}\right) = 10e^{0.4x}$$ M1

Giving

$$y = \frac{10e^{0.4x}}{9 + e^{0.4x}} = \frac{10}{1 + 9e^{-0.4x}}$$ A1

2. Use separation of variables M1

$$\int \frac{1}{y^2+1}dy = \int 1\,dx$$ A1

So

$$\arctan y = x + c$$ A1

Giving

$$y = \tan x + C$$ A1

Since $y(0) = 0$ then $C = 0$. M1A1

So $y = \tan x$.