1. If n is an even integer then $7n+4$ is an even integer.

Let $n = 2k, k \in \mathbb{Z}$.

$$7n+4 = 7(2k)+4$$
$$= 2(7k+2)$$

$7k+2 \in \mathbb{Z}$ $\therefore$ $7n+4$ is an even integer.

2. If m is an even integer and n is an odd integer then $m+n$ is an odd integer.

Let $m = 2k$ and $n = 2p+1$, $m, n, k, p \in \mathbb{Z}$.

$$m+n = 2k+2p+1$$
$$= 2(k+p)+1$$

$k+p \in \mathbb{Z}$ $\therefore$ $m+n$ is an odd integer

3. If m is an even integer and n is an odd integer, then mn is an even integer.

Let $m = 2k$, $n = 2p+1$, $mn, k, p \in \mathbb{Z}$.

$$m \cdot n = 2k \cdot (2p+1)$$
$$= 2[k(2p+1)]$$

$k(2p+1) \in \mathbb{Z}$ $\therefore$ $m \cdot n$ is an even integer.

1. If a, b and c are integers such that a divides b and b divides c, then a divides c.

Let $b = ka$ and $c = pb$, $k, p \in \mathbb{Z}$.

$b = \frac{c}{p}$; $\frac{c}{p} = ka$; $c = pk \cdot a$

Since $p \cdot k \in \mathbb{Z}$ $\therefore$ a divides c exactly.