

a) Calculate the integrals:

$$1. \int \sin x \cos x \, dx$$

$$3. \int \cos^2 x \, dx$$

$$2. \int \frac{(x+1)^2}{2x(1+x^2)} \, dx$$

$$4. \int \sin^2 x \, dx$$

SOLUTIONS

$$\begin{aligned} 1. \int \sin x \cos x \, dx &= \int \sin 2x \, dx \\ &= -\frac{1}{2} \cos 2x + C \end{aligned}$$

$$\begin{aligned} 2. \int \frac{(x+1)^2}{2x(1+x^2)} \, dx &= \int \frac{x^2+2x+1}{2x(1+x^2)} \, dx \\ &= \int \frac{x^2+1+2x}{2x(1+x^2)} \, dx = \int \left[ \frac{x^2+1}{2x(1+x^2)} + \frac{2x}{2x(1+x^2)} \right] \, dx \\ &= \int \frac{x^2+1}{2x(1+x^2)} \, dx + \int \frac{2x}{2x(1+x^2)} \, dx \\ &= \int \frac{1}{2x} \, dx + \int \frac{1}{1+x^2} \, dx \\ &= \frac{1}{2} \ln|x| + \arctan x + C \end{aligned}$$

$$\begin{aligned} 3. \int \cos^2 x \, dx &= \int \frac{1 + \cos 2x}{2} \, dx \\ &= \int \left[ \frac{1}{2} + \frac{1}{2} \cos 2x \right] \, dx = \int \frac{1}{2} \, dx + \frac{1}{2} \int \cos 2x \, dx \\ &= \frac{1}{2} x + \frac{1}{4} \sin 2x + C \end{aligned}$$

$$\begin{aligned} 4. \int \sin^2 x \, dx &= \int \frac{1 - \cos 2x}{2} \, dx \\ &= \int \frac{1}{2} \, dx - \frac{1}{2} \int \cos 2x \, dx \\ &= \frac{1}{2} x - \frac{1}{4} \sin 2x + C \end{aligned}$$