2 Estimation of π — Type I

Work done on this assignment will be assessed against criteria A, B, C, D and F. You are therefore expected to use a graphic display calculator and/or computer for this assignment.

The goal of this assignment is to estimate the value of π through finding the area of the unit circle. To find an estimate for this area you need to calculate the areas of n-sided inscribed and circumscribed polygons.

This method was used by the Greek mathematician Archimedes over two thousand years ago.

1 Explain in simple terms why the value of π lies between the areas of the two polygons described above.

2 Develop a general formula for the area of an n-sided equilateral polygon which is inscribed in a circle of radius r.

Hint: Draw a sketch of a typical triangular piece of the n-sided polygon.

3 Develop a general formula for the area of an n-sided equilateral polygon which is circumscribed in a circle of radius r.

4 Using $r = 1$, and n from 10 to 400 (in steps of 10), calculate the values of the areas of the inscribed and circumscribed polygons.

(a) For each polygon how many decimal places of accuracy are there when $n = 400$? Note that $\pi = 3.141592654$ to 9 decimal places.

(b) Calculate the average of the areas of the inscribed and circumscribed polygons for $n = 400$. How many places of accuracy are there?

5 What value of n is necessary to obtain six decimal places of accuracy in both approximations, given that $n \geq 400$.

6 Plot the values of the areas you have found against the number of sides. Also label the value of π. Which approximation converges faster? Try to justify your answer.

7 Archimedes also found a value for π using the semi-perimeter of the inscribed and circumscribed polygons. Using this alternative method, follow a similar procedure to the one described previously to find an estimate for the value of π.