1. PROVE THAT $(a+b)^2+(a-b)^2=2(a^2+b^2)$

$$(a+b)^2+(a-b)^2 = a^2+2ab+b^2+a^2-2ab+b^2$$
$$=2a^2+2b^2$$
$$=2(a^2+b^2)$$

2. THE PRODUCT OF TWO ODD NUMBERS IS ODD.

let $2k+1$, $2p+1$, $k,p\in\mathbb{Z}$

then $(2k+1)(2p+1) = 2k\cdot 2p+2k+2p+1$
$= 2(2kp+k+p)+1$
$\because$ $2kp+k+p\in\mathbb{Z}$
$\therefore$ the product is odd

3. Let abcd a 4-digit number $n\in\mathbb{Z}$

then $n = 1000a+100b+10c+d$ ①

IT'S GIVEN THAT $a+b+c+d=9p$, $p\in\mathbb{Z}$

then $d=9p-a-b-c$. ②.

SUBSTITUTE ② in ①

$$n = 1000a+100b+10c+(9p-a-b-c)$$
$$= 999a+99b+9c+9p$$
$$= 9(111a+11b+c+p)$$

$\because$ $111a+11b+c+p\in\mathbb{Z}$
$\therefore$ n is divisible by 9.

4. $(a^2+b^2)(c^2+d^2) = a^2c^2+a^2d^2+b^2c^2+b^2d^2$
$= a^2d^2+b^2c^2 + a^2c^2+b^2d^2$
$= a^2d^2+b^2c^2+2abcd+a^2c^2+b^2d^2-2abcd$
$= (ad+bc)^2+(bd-ac)^2$

5. $\frac{1}{3}-\frac{2}{9}+\frac{1}{27}-\frac{2}{81}+\frac{1}{243}-\frac{2}{729}+\dots = \underbrace{\frac{1}{3}+\frac{1}{27}+\frac{1}{243}+\dots}_{G.S\ r=\frac{1}{9},\ a_1=\frac{1}{3}} - \underbrace{\left(\frac{2}{9}+\frac{2}{81}+\frac{2}{729}\right)}_{G.S.\ r=\frac{1}{9},\ a_1=\frac{2}{9}}$

$$= \frac{\frac{1}{3}}{1-\frac{1}{9}} - \frac{\frac{2}{9}}{1-\frac{1}{9}} = \frac{\frac{1}{3}}{\frac{8}{9}} - \frac{\frac{2}{9}}{\frac{8}{9}} = \frac{3}{8}-\frac{1}{4}$$
$$=\frac{1}{8}.$$

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6. let $2n$, $2n+1$, $n \in \mathbb{Z}$ two consecutive numbers.

then $(2n+1)^2 - (2n)^2 = 4n^2 + 4n + 1 - 4n^2$

$$= 4n + 1$$

$$= 2(2n) + 1$$

$\because$ $2n \in \mathbb{Z}$

$\therefore$ the difference is odd.

7. $$\frac{1}{n-1} - \frac{1}{n} + \frac{1}{n+1} = \frac{n(n+1) - (n-1)(n+1) + n(n-1)}{n(n-1)(n+1)}$$

$$= \frac{\cancel{n^2} + \cancel{n} - \cancel{n^2} + 1 + n^2 - \cancel{n}}{n(n^2-1)}$$

$$= \frac{n^2+1}{n(n^2-1)}$$

then $\frac{1}{5} - \frac{1}{6} + \frac{1}{7} = \frac{6^2+1}{6(6^2-1)} = \frac{37}{6(35)} = \frac{37}{210}$