TEST 3 Study guide solutions

1. $1-\sqrt{3}i \Rightarrow r=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{4}=2$ $\therefore\ 1-\sqrt{3}i = 2\,\mathrm{Cis}\left(\frac{10\pi}{6}\right)$

$\theta = \mathrm{Tan}^{-1}\left(-\frac{\sqrt{3}}{1}\right)=\frac{10\pi}{6}$

$(1-\sqrt{3}i)^3 = \left(2\,\mathrm{Cis}\left(\frac{10\pi}{6}\right)\right)^3 = 8\,\mathrm{Cis}^3\left(\frac{10\pi}{6}\right)$. Since $\arg(z^n)=n\arg(z)$

$= 8\,\mathrm{Cis}\left(\frac{30\pi}{6}\right) = 8\,\mathrm{Cis}(5\pi) = 8(\mathrm{cis}\,5\pi + i\sin 5\pi)$

$= -8$

$\therefore\ \frac{1}{(1-\sqrt{3}i)^3} = -\frac{1}{8}$.

2. $iz+2 = i-3z$

$iz+3z = i-2$

$z(i+3) = i-2$

$z = \frac{i-2}{i+3}\cdot\frac{-i+3}{-i+3}$

$= \frac{1+3i+2i-6}{4}$

$= -\frac{5}{4}+\frac{5}{4}i$

3. $(1+ai)(1+bi) = b+9i-a$

$1+ai+bi-ab = b-a+9i$

$1-ab + i(a+b) = b-a+9i$

$\therefore\ 1-ab = b-a$

$a+b=9$

$b = 9-a$

$1-a(9-a) = 9-a-a$

$a^2-7a-8=0$

$(a-8)(a+1)=0$

$a=8 \Rightarrow b=1$

OR

$a=-1 \Rightarrow b=10$

$(1-2i)^2 = -3-4i$

4. a) $(a+ib)^2 = -3-4i$

$a^2-b^2+2iba = -3-4i$

$\therefore\ a^2-b^2=-3$

$2ab=-4$

$b=-\frac{2}{a}$

$a^2-\frac{4}{a^2}=-3$

$a^4+3a^2-4=0$

$(a^2+4)(a^2-1)=0$

$\therefore\ a=\pm 1$

if $a=1$, $b=-2$

if $a=-1$, $b=2$

b) $z^2+i\sqrt{3}z+i=0$

$z^2+i\sqrt{3}z+\left(\frac{i\sqrt{3}}{2}\right)^2-\left(\frac{i\sqrt{3}}{2}\right)^2+i=0$

$\left(z+\frac{i\sqrt{3}}{2}\right)^2+\frac{3}{4}+i=0$

$\left(z+\frac{i\sqrt{3}}{2}\right)^2 = \frac{-3-4i}{4}$

$(2z+i\sqrt{3})^2 = -3-4i$

$\therefore\ 2z+i\sqrt{3} = 1-2i$

$\therefore\ z=\frac{1}{2}-\frac{(2+\sqrt{3})i}{2}$

OR $2z+i\sqrt{3} = -1+2i$

$\therefore\ z=-\frac{1}{2}+\frac{(2-\sqrt{3})i}{2}$

5. a) $3\,\mathrm{Cis}\left(\frac{7\pi}{4}\right) = 3\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$= 3\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$

$= \frac{3\sqrt{2}}{2} - i\frac{3\sqrt{2}}{2}$

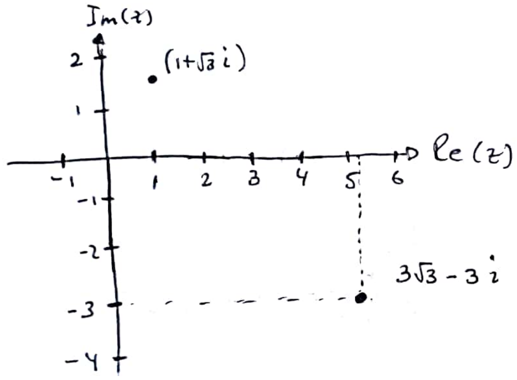
b) $4i - 4 = 4\sqrt{2}\,\mathrm{Cis}\left(\frac{7\pi}{4}\right)$

6. a) $z = 1 + \sqrt{3}i$ and $w = 3\sqrt{3} - 3i = 3(\sqrt{3} - i)$

$|z| = 2$ $\quad |w| = 6$

$\arg(z) = \frac{\pi}{3}$ $\quad \arg(w) = \frac{11\pi}{6}$

b)



c) $|zw| = |z|\cdot|w| = 12$

$\arg(zw) = \arg(z) + \arg(w)$

$= \frac{\pi}{3} + \frac{11\pi}{6} = \frac{13\pi}{6}$

$= \frac{\pi}{6}$

7. $2z^* + \frac{3}{iz} = \sqrt{15}$ $\quad \times z$

$2z^*\cdot z + \frac{3}{i} = \sqrt{15}\,z$

$2|z|^2 - 3i = \sqrt{15}\,z$; $|z| = \sqrt{3}$

$\frac{2(3) - 3i}{\sqrt{15}} = z$

$\therefore\ z = \frac{6 - 3i}{\sqrt{15}}$

8. $z = x + iy$

$$\frac{z}{z+1} = \frac{x+iy}{x+1+iy} \cdot \frac{(x+1)-iy}{(x+1)-iy}.$$

$$= \frac{x(x+1) - \cancel{ixy} + i\cancel{(x)}y + iy + y^2}{(x+1)^2 + y^2}$$

$$= \frac{x^2 + x + iy + y^2}{(x+1)^2 + y^2}$$

$$= \frac{x(x+1)}{(x+1)^2+y^2} + i\,\frac{y(y+1)}{(x+1)^2+y^2}$$

$$\therefore \ Re(z/(z+1)) = \frac{x(x+1)}{(x+1)^2+y^2}$$

$$Im(z/(z+1)) = \frac{y(y+1)}{(x+1)^2+y^2}$$

9. a) $-\sqrt{2} + \sqrt{2}\,i = \sqrt{2}\,(-1+i)$

$= 2\,Cis\left(\frac{3\pi}{4}\right)$

b) $(-\sqrt{2} + \sqrt{2}\,i)^6 = \left(2\,Cis\,\frac{3\pi}{4}\right)^6$

$= 2^6\,Cis\left(\frac{18\pi}{4}\right)$

$= 2^6\,Cis\left(\frac{9\pi}{2}\right)$

$= 2^6 = 64$

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